**REMOVAL OF OUTLIERS ANALYSIS**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Pre-1982.0** | | | | |
| **Number of outliers removal** | Original | 1 | 2 | 3 | 4 |
| **Mean (g)** | 3.0837 | 3.0891 | 3.0892 | 3.0892 | 3.0892 |
| **Standard deviation (g)** | 0.0867 | 0.0334 | 0.0306 | 0.0301 | 0.0300 |
| **Minimum value (g)** | 2.0152 | 2.9276 | 2.9935 | 2.9978 | 2.9997 |
| **Maximum value (g)** | 3.9652 | 3.2625 | 3.1849 | 3.1749 | 3.1749 |
| **Relative standard deviation (ppt)** | 28.1043 | 10.8246 | 9.9181 | 9.7336 | 9.7122 |
| **3 σ (g)** | 0.2600 | 0.1003 | 0.0919 | 0.0902 | 0.0900 |
| **Mean + 3 σ (g)** | 3.3437 | 3.1894 | 3.1811 | 3.1794 | 3.1792 |
| **Mean - 3 σ (g)** | 2.8237 | 2.9888 | 2.9973 | 2.9990 | 2.9992 |

Table 1: Outliers removal of pre-1982.0 group

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Post-1982.5** | | | | | |
| **Number of outliers removal** | Original | 1 | 2 | 3 | 4 | 5 |
| **Mean (g)** | 2.5077 | 2.5053 | 2.5048 | 2.5046 | 2.5046 | 2.5046 |
| **Standard deviation (g)** | 0.0431 | 0.0210 | 0.0195 | 0.0193 | 0.0191 | 0.0191 |
| **Minimum value (g)** | 2.4083 | 2.4083 | 2.4467 | 2.4467 | 2.4480 | 2.4480 |
| **Maximum value (g)** | 3.5135 | 2.5976 | 2.5656 | 2.5629 | 2.5621 | 2.5616 |
| **Relative standard deviation (ppt)** | 17.2073 | 8.3958 | 7.7665 | 7.6899 | 7.6317 | 7.6174 |
| **3 σ (g)** | 0.1294 | 0.0631 | 0.0584 | 0.0578 | 0.0573 | 0.0572 |
| **Mean + 3 σ (g)** | 2.6371 | 2.5684 | 2.5631 | 2.5624 | 2.5619 | 2.5618 |
| **Mean - 3 σ (g)** | 2.3782 | 2.4422 | 2.4464 | 2.4469 | 2.4472 | 2.4473 |

Table 2: Outliers removal of post-1982.5 group

|  |  |  |
| --- | --- | --- |
|  | **Final removal of outliers** | |
| **Group** | Pre-1982.0 | Post-1982.5 |
| **Mean (g)** | 3.0892 | 2.5046 |
| **Standard deviation (g)** | 0.0300 | 0.0191 |
| **Minimum value (g)** | 2.9997 | 2.4480 |
| **Maximum value (g)** | 3.1749 | 2.5616 |
| **Relative standard deviation (ppt)** | 9.7122 | 7.6174 |
| **3 σ (g)** | 0.0900 | 0.0572 |
| **Mean + 3 σ (g)** | 3.1792 | 2.5618 |
| **Mean - 3 σ (g)** | 2.9992 | 2.4473 |

Table 3: Final removal of outliers for both group

**YEAR-BY-YEAR ANALYSIS**

|  |  |  |
| --- | --- | --- |
| **Pre-1982.0** | | |
| **Year** | **Mean (g)** | **Standard deviation (g)** |
| **1950** | 3.0881 | 0.0000 |
| **1956** | 3.0883 | 0.0001 |
| **1959** | 3.1056 | 0.0168 |
| **1960** | 3.0994 | 0.0000 |
| **1961** | 3.0957 | 0.0230 |
| **1962** | 3.1035 | 0.0333 |
| **1963** | 3.1003 | 0.0453 |
| **1964** | 3.0980 | 0.0366 |
| **1965** | 3.0926 | 0.0211 |
| **1966** | 3.0942 | 0.0166 |
| **1967** | 3.0848 | 0.0265 |
| **1968** | 3.0707 | 0.0281 |
| **1969** | 3.0936 | 0.0341 |
| **1970** | 3.1021 | 0.0358 |
| **1971** | 3.1070 | 0.0429 |
| **1972** | 3.0820 | 0.0209 |
| **1973** | 3.0755 | 0.0305 |
| **1974** | 3.0889 | 0.0368 |
| **1975** | 3.0967 | 0.0241 |
| **1976** | 3.0876 | 0.0296 |
| **1977** | 3.0833 | 0.0297 |
| **1978** | 3.0920 | 0.0263 |
| **1979** | 3.0846 | 0.0265 |
| **1980** | 3.0876 | 0.0267 |
| **1981** | 3.0923 | 0.0266 |
| **1982.0** | 3.0839 | 0.0266 |

|  |  |  |
| --- | --- | --- |
| **Post-1982.5** | | |
| **Year** | **Mean (g)** | **Standard deviation (g)** |
| **1982.5** | 2.5149 | 0.0244 |
| **1983** | 2.5189 | 0.0253 |
| **1984** | 2.5170 | 0.0210 |
| **1985** | 2.5246 | 0.0192 |
| **1986** | 2.5192 | 0.0238 |
| **1987** | 2.5031 | 0.0199 |
| **1988** | 2.5011 | 0.0217 |
| **1989** | 2.5179 | 0.0190 |
| **1990** | 2.5076 | 0.0203 |
| **1991** | 2.5053 | 0.0204 |
| **1992** | 2.5016 | 0.0235 |
| **1993** | 2.5010 | 0.0152 |
| **1994** | 2.5027 | 0.0124 |
| **1995** | 2.4975 | 0.0142 |
| **1996** | 2.5016 | 0.0186 |
| **1997** | 2.4924 | 0.0173 |
| **1998** | 2.5024 | 0.0138 |
| **1999** | 2.5019 | 0.0141 |
| **2000** | 2.5022 | 0.0111 |
| **2001** | 2.5040 | 0.0144 |
| **2002** | 2.4996 | 0.0112 |
| **2003** | 2.4983 | 0.0171 |
| **2004** | 2.5016 | 0.0136 |
| **2005** | 2.5023 | 0.0197 |
| **2006** | 2.4993 | 0.0183 |
| **2007** | 2.4984 | 0.0148 |
| **2008** | 2.5060 | 0.0166 |
| **2009** | 2.4961 | 0.0162 |
| **2010** | 2.4978 | 0.0140 |
| **2012** | 2.5155 | 0.0000 |
| **2013** | 2.4938 | 0.0100 |

Table 4A & 4B: Average mass of pennies each year, for pre-1982.0 and post-1982.5 group respectively

Figure 1: Graph of US One Cent coins from 1950 to the present year

Figure 2: US One Cent coins from 1982.5 to 2013, excluding 2009

|  |  |  |  |
| --- | --- | --- | --- |
| **LINEST POST-1982.5** | | | |
| **m** | -0.000671 | 3.844721 | **b** |
| **μm** | 0.000051 | 0.101141 | **μb** |
| **R2** | 0.076542 | 0.018349 | **sy** |
| **Fisher value** | 175.552214 | 2118.000000 | **df** |
| **Regression ss** | 0.059108 | 0.713129 | **Residual ss** |

Table 5: LINEST value of post-1982.5 group

|  |  |
| --- | --- |
| **POST-1982.5** | |
| **σm or μm (g/year)** | 0.000051 |
| **m (g/year)** | -0.000671 |
| **m + μm (g/year)** | -0.000621 |
| **m - μm (g/year)** | -0.000722 |

Table 6: Comparison of mean with its standard error

|  |  |
| --- | --- |
| **POST-1982.5** | |
| **σm or μm (g/year)** | 0.000051 |
| **m (g/year)** | -0.000671 |
| **t(95% CI, df ∞)** | 1.960000 |
| **t\*μm (g/year)** | 0.000099 |
| **m + t\*μm (g/year)** | -0.000572 |
| **m - t\*μm (g/year)** | -0.000771 |

Table 7: Comparison of means with the standard error by student’s t-test

Figure 3: US One Cent coin of 1963 to 1982.0

|  |  |  |  |
| --- | --- | --- | --- |
| **LINEST PRE-1982.0** | | | |
| **m** | -0.000355 | 3.791006 | **b** |
| **μm** | 0.000136 | 0.269262 | **μb** |
| **R2** | 0.003653 | 0.030003 | **sy** |
| **Fisher value** | 6.796654 | 1854.000000 | **df** |
| **Regression ss** | 0.006118 | 1.668880 | **Residual ss** |

Table 8; LINEST value for pre-1982.0 group

|  |  |
| --- | --- |
| **PRE-1982.0** | |
| **σm or μm (g/year)** | 0.000136 |
| **m (g/year)** | -0.000355 |
| **m + μm (g/year)** | -0.000219 |
| **m - μm (g/year)** | -0.000492 |

Table 9: Comparison of means with its standard error

|  |  |
| --- | --- |
| **PRE-1982.0** | |
| **σm or μm (g/year)** | 0.000136 |
| **m (g/year)** | -0.000355 |
| **t(95% CI, df ∞)** | 1.960000 |
| **t\*μm (g/year)** | 0.000267 |
| **m + t\*μm (g/year)** | -0.000088 |
| **m - t\*μm (g/year)** | -0.000622 |

Table 10: Comparison of means with its standard error by student’s t-test

**YEAR-BY-YEAR ANALYSIS – F-TEST AND T-TEST**

|  |  |
| --- | --- |
| **Post-1982.5** | |
| **Highest average mass (g)** | 2.5246 |
| **Lowest average mass (g)** | 2.4924 |

Table 11: Highest and lowest average mass of post-1982.5 group

|  |  |  |
| --- | --- | --- |
| **Post-1982.5** | | |
| **F-Test Two-Sample for Variances** | | |
| **Year** | **1985** | **1997** |
| **Mean (g)** | 2.5246 | 2.4924 |
| **Variance (g2)** | 0.0004 | 0.0003 |
| **Observations** | 71 | 42 |
| **df** | 70 | 41 |
| **F** | 1.2400 | |
| **P(F<=f) one-tail** | 0.2303 | |
| **F Critical one-tail** | 1.6129 | |
| **F-test PASS** | | |

Table 12: F-test for post-1982.5 group

|  |  |  |
| --- | --- | --- |
| **Post-1982.5** | | |
| **t-Test: Two-Sample Assuming Equal Variances** | | |
| **Year** | **1985** | **1997** |
| **Mean (g)** | 2.5246 | 2.4924 |
| **Variance (g2)** | 0.0004 | 0.0003 |
| **Observations** | 71 | 42 |
| **Pooled Variance** | 0.0003 | |
| **Hypothesized Mean Difference** | 0 | |
| **df** | 111 | |
| **t Stat** | 8.9355 | |
| **P(T<=t) one-tail** | 0.0000 | |
| **t Critical one-tail** | 1.6587 | |
| **P(T<=t) two-tail** | 0.0000 | |
| **t Critical two-tail** | 1.9816 | |
| **t-test FAIL** | | |

Table 13: t-test of post-1982.5 group

|  |  |
| --- | --- |
| **Pre-1982.0** | |
| **Highest average mass (g)** | 3.1070 |
| **Lowest average mass (g)** | 3.0707 |

Table 14: Highest and lowest average mass of pre-1982.0 group

|  |  |  |
| --- | --- | --- |
| **Pre-1982.0** | | |
| **F-Test Two-Sample for Variances** | | |
| **Year** | **1971** | **1968** |
| **Mean (g)** | 3.1070 | 3.0707 |
| **Variance (g2)** | 0.0018 | 0.0008 |
| **Observations** | 66 | 57 |
| **df** | 65 | 56 |
| **F** | 2.3248 | |
| **P(F<=f) one-tail** | 0.0008 | |
| **F Critical one-tail** | 1.5395 | |
| **F-test FAIL** | | |

Table 15: F-test of pre-1982.0 group

|  |  |  |
| --- | --- | --- |
| **Pre-1982.0** | | |
| **t-Test: Two-Sample Assuming Unequal Variances** | | |
| **Year** | **1971** | **1968** |
| **Mean (g)** | 3.1070 | 3.0707 |
| **Variance (g2)** | 0.0018 | 0.0008 |
| **Observations** | 66 | 57 |
| **Hypothesized Mean Difference** | 0 | |
| **df** | 113 | |
| **t Stat** | 5.6190 | |
| **P(T<=t) one-tail** | 0.0000 | |
| **t Critical one-tail** | 1.6585 | |
| **P(T<=t) two-tail** | 0.0000 | |
| **t Critical two-tail** | 1.9812 | |
| **t-test FAIL** | | |

Table 16: t-test of pre-1982.0 group

**COMPARISON BY MINT LOCATION**

|  |  |  |
| --- | --- | --- |
| **Post-1982.5** | | |
| **Mint Location** | P | D |
| **Mean (g)** | 2.5047 | 2.5046 |
| **Standard deviation (g)** | 0.0191 | 0.0190 |
| **Minimum value (g)** | 2.4502 | 2.4480 |
| **Maximum value (g)** | 2.5616 | 2.5526 |

Table 17: Data for two different minted locations

|  |  |  |
| --- | --- | --- |
| **Post-1982.5** | | |
| **F-Test Two-Sample for Variances** | | |
| **Mint Location** | P | D |
| **Mean (g)** | 2.5047 | 2.5046 |
| **Variance (g2)** | 0.0004 | 0.0004 |
| **Observations** | 1967 | 153 |
| **df** | 1966 | 152 |
| **F** | 1.0080 | |
| **P(F<=f) one-tail** | 0.4868 | |
| **F Critical one-tail** | 1.2286 | |
| **F-test PASS** | | |

Table 18: F-test of two different minted locations for post-1982.5 group

|  |  |  |
| --- | --- | --- |
| **Post-1982.5** | | |
| **t-Test: Two-Sample Assuming Equal Variances** | | |
| **Mint Location** | P | D |
| **Mean (g)** | 2.5047 | 2.5046 |
| **Variance (g2)** | 0.0004 | 0.0004 |
| **Observations** | 1967 | 153 |
| **Pooled Variance** | 0.0004 | |
| **Hypothesized Mean Difference** | 0 | |
| **df** | 2118 | |
| **t Stat** | 0.0238 | |
| **P(T<=t) one-tail** | 0.4905 | |
| **t Critical one-tail** | 1.6456 | |
| **P(T<=t) two-tail** | 0.9810 | |
| **t Critical two-tail** | 1.9611 | |
| **t-test PASS** | | |

Table 19: t-test of two different minted locations for post-1982.5 group

|  |  |  |
| --- | --- | --- |
| **Pre-1982.0** | | |
| **Mint Location** | D | P |
| **Mean (g)** | 3.0933 | 3.0884 |
| **Standard deviation (g)** | 0.0327 | 0.0296 |
| **Minimum value (g)** | 2.9997 | 3.0009 |
| **Maximum value (g)** | 3.1749 | 3.1742 |

Table 20: Data for pre-1982.0 group

|  |  |  |
| --- | --- | --- |
| **Pre-1982.0** | | |
| **F-Test Two-Sample for Variances** | | |
| **Mint Location** | D | P |
| **Mean (g)** | 3.0933 | 3.0884 |
| **Variance (g2)** | 0.0011 | 0.0009 |
| **Observations** | 244 | 1613 |
| **df** | 243 | 1612 |
| **F** | 1.2230 | |
| **P(F<=f) one-tail** | 0.0160 | |
| **F Critical one-tail** | 1.1675 | |
| **F-test FAIL** | | |

Table 21: F-test for two different minted locations for pre-1982.0 group

|  |  |  |
| --- | --- | --- |
| **Pre-1982.0** | | |
| **t-Test: Two-Sample Assuming Unequal Variances** | | |
| **Mint Location** | D | P |
| **Mean (g)** | 3.0933 | 3.0884 |
| **Variance (g2)** | 0.0011 | 0.0009 |
| **Observations** | 244 | 1613 |
| **Hypothesized Mean Difference** | 0 | |
| **df** | 306 | |
| **t Stat** | 2.2064 | |
| **P(T<=t) one-tail** | 0.0141 | |
| **t Critical one-tail** | 1.6498 | |
| **P(T<=t) two-tail** | 0.0281 | |
| **t Critical two-tail** | 1.9677 | |
| **t-test PASS** | | |

Table 22: t-test for two different minted locations for pre-1982.0 group

**DISTRIBUTION OF MASS ANALYSIS**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **Average (g)** | **Standard Deviation (g)** | **Minimum value (g)** | **Maximum value (g)** |
| 1982.5 | 2.5149 | 0.0244 | 2.4590 | 2.5504 |
| 1983 | 2.5189 | 0.0253 | 2.4508 | 2.5616 |
| 1984 | 2.5170 | 0.0210 | 2.4710 | 2.5592 |
| 1985 | 2.5246 | 0.0192 | 2.4715 | 2.5561 |
| 1986 | 2.5192 | 0.0238 | 2.4629 | 2.5536 |
| 1987 | 2.5031 | 0.0199 | 2.4480 | 2.5562 |
| 1988 | 2.5011 | 0.0217 | 2.4506 | 2.5471 |
| 1989 | 2.5179 | 0.0190 | 2.4660 | 2.5609 |
| 1990 | 2.5076 | 0.0203 | 2.4626 | 2.5498 |
| 1991 | 2.5053 | 0.0204 | 2.4680 | 2.5492 |
| 1992 | 2.5016 | 0.0235 | 2.4502 | 2.5557 |
| 1993 | 2.5010 | 0.0152 | 2.4612 | 2.5528 |
| 1994 | 2.5027 | 0.0124 | 2.4755 | 2.5446 |
| 1995 | 2.4975 | 0.0142 | 2.4648 | 2.5563 |
| 1996 | 2.5016 | 0.0186 | 2.4644 | 2.5604 |
| 1997 | 2.4924 | 0.0173 | 2.4640 | 2.5305 |
| 1998 | 2.5024 | 0.0138 | 2.4753 | 2.5374 |
| 1999 | 2.5019 | 0.0141 | 2.4658 | 2.5458 |
| 2000 | 2.5022 | 0.0111 | 2.4688 | 2.5405 |
| 2001 | 2.5040 | 0.0144 | 2.4703 | 2.5513 |
| 2002 | 2.4996 | 0.0112 | 2.4723 | 2.5213 |
| 2003 | 2.4983 | 0.0171 | 2.4611 | 2.5603 |
| 2004 | 2.5016 | 0.0136 | 2.4686 | 2.5529 |
| 2005 | 2.5023 | 0.0197 | 2.4576 | 2.5340 |
| 2006 | 2.4993 | 0.0183 | 2.4531 | 2.5543 |
| 2007 | 2.4984 | 0.0148 | 2.4618 | 2.5323 |
| 2008 | 2.5060 | 0.0166 | 2.4766 | 2.5381 |
| 2010 | 2.4978 | 0.0140 | 2.4659 | 2.5238 |
| 2012 | 2.5155 | 0.0000 | 2.5155 | 2.5155 |
| 2013 | 2.4938 | 0.0100 | 2.4791 | 2.5083 |

Table 23: The data for post-1982.5 group (excluding data from 2009)

|  |  |  |
| --- | --- | --- |
| **Mass range (g)** | **Mass of penny (g)** | **Frequency** |
| **2.40-2.41** | 2.4100 | 0 |
| **2.41-2.42** | 2.4200 | 0 |
| **2.42-2.43** | 2.4300 | 0 |
| **2.43-2.44** | 2.4400 | 0 |
| **2.44-2.45** | 2.4500 | 1 |
| **2.45-2.46** | 2.4600 | 13 |
| **2.46-2.47** | 2.4700 | 47 |
| **2.47-2.48** | 2.4800 | 122 |
| **2.48-2.49** | 2.4900 | 251 |
| **2.49-2.50** | 2.5000 | 464 |
| **2.50-2.51** | 2.5100 | 472 |
| **2.51-2.52** | 2.5200 | 348 |
| **2.52-2.53** | 2.5300 | 186 |
| **2.53-2.54** | 2.5400 | 108 |
| **2.54-2.55** | 2.5500 | 67 |
| **2.55-2.56** | 2.5600 | 36 |
| **2.56-2.57** | 2.5700 | 5 |
| **2.57-2.58** | 2.5800 | 0 |
| **2.58-2.59** | 2.5900 | 0 |
| **2.59-2.60** | 2.6000 | 0 |

Table 24: Frequency table of the mass of pennies in the post-1982.5 group

Figure 4: Histogram of mass of pennies from 1982.5-2013, excluding 2009